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PROJECTIVE CURVATURE TENSOR OF A SEMI-SYMMETRIC METRIC CONNECTION IN A KENMOTSU MANIFOLD

AJIT BARMAN AND U. C. DE

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Abstract. The object of the present paper is to study a Kenmotsu manifold admitting a semi-symmetric metric connection whose projective curvature tensor satisfies certain curvature conditions.

1. Introduction

The product of an almost contact manifold M and the real line R carries a natural almost complex structure. However if one takes M to be an almost contact metric manifold and suppose that the product metric G on $M \times R$ is Kaehlerian, then the structure on M is cosymplectic [12] and not Sasakian. On the other hand Oubina [15] pointed out that if the conformally related metric $e^{2t}G$, t being the coordinate on R, is Kaehlerian, then M is Sasakian and conversely.

In [19], S. Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold M, the sectional curvature of plane sections containing ξ is a constant, say c. If $c > 0$, M is a homogeneous Sasakian manifold of constant sectional curvature. If $c = 0$, M is the product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. If $c < 0$, M is a warped product space $R \times_f C^n$. In 1971, Kenmotsu studied a class of contact Riemannian manifolds satisfying some special conditions [14]. We call it Kenmotsu manifold. Kenmotsu manifolds have been studied by J.B. Jun , U.C. De and G. Pathak [13], C. Özgür and U.C. De [16], U.C. De and G. Pathak [9], A. Yıldiz, U.C. De and B.E. Acet [22] and others.

H.A. Hayden [11] introduced semi-symmetric linear connections on a Riemannian manifold and this was further developed by K. Yano [20], K. Amur and S.S. Pujar [1], M. Prvanović [17], U.C. De and S.C. Biswas [8], A. Sharfuddin and S.I. Hussain [18], T.Q. Binh [3], F.Ö. Zengin and S.A. Uysal and S.A. Demirbag [26], S.K. Chaubey and R.H. Ojha ([6], [7]), H.B. Yılmaz [23] and others.

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Let M be an n-dimensional Riemannian manifold of class C^∞ endowed with the Riemannian metric g and D be the Levi-Civita connection on (M^n, g).

A linear connection ∇ defined on (M^n, g) is said to be semi-symmetric [10] if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y, \tag{1.1}$$

where η is a 1-form and ξ is a vector field given by

$$\eta(X) = g(X, \xi), \tag{1.2}$$

for all vector fields $X \in \chi(M^n)$, $\chi(M^n)$ is the set of all differentiable vector fields on M^n.

A semi-symmetric connection ∇ is called a semi-symmetric metric connection [11] if it further satisfies

$$\nabla g = 0. \tag{1.3}$$

A relation between the semi-symmetric metric connection ∇ and the Levi-Civita connection D on (M^n, g) has been obtained by K. Yano [20] which is given by

$$\nabla_X Y = D_X Y + \eta(Y)X - g(X,Y)\xi. \tag{1.4}$$

We also have

$$(\nabla_X \eta)(Y) = (D_X \eta)Y - \eta(X)\eta(Y) + \eta(\xi)g(X,Y). \tag{1.5}$$

Further, a relation between the curvature tensor R of the semi-symmetric metric connection ∇ and the curvature tensor K of the Levi-Civita connection D is given by

$$\begin{aligned} R(X,Y)Z = K(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X + \\ g(X,Z)QY - g(Y,Z)QX, \end{aligned} \tag{1.6}$$

where α is a tensor field of type (0,2) and Q is a tensor field of type (1,1) which is given by

$$\alpha(Y,Z) = g(QY,Z) = (D_Y \eta)(Z) - \eta(Y)\eta(Z) + \frac{1}{2}\eta(\xi)g(Y,Z). \tag{1.7}$$

From (1.6) and (1.7), we obtain

$$\begin{aligned} \tilde{R}(X,Y,Z,W) = \tilde{K}(X,Y,Z,W) - \alpha(Y,Z)g(X,W) + \\ \alpha(X,Z)g(Y,W) - g(Y,Z)\alpha(X,W) + \\ g(X,Z)\alpha(Y,W), \end{aligned} \tag{1.8}$$

where

$$\tilde{R}(X,Y,Z,W) = g(R(X,Y)Z,W), \quad \tilde{K}(X,Y,Z,W) = g(K(X,Y)Z,W). \tag{1.9}$$

The Projective curvature tensor is an important tensor from the differential geometric point of view. Let M be a $(2n+1)$-dimensional Riemannian manifold. If there exists a one-to-one correspondence between each coordinate neighbourhood of M and a domain in Euclidian space such that any geodesic of the Riemannian manifold corresponds to a straight line in the Euclidean space, then M is said to be locally projectively flat. For $n \geq 1$, M is locally projectively flat if and only if the projective curvature tensor P vanishes. Here the projective curvature tensor P with respect to the semi-symmetric metric connection is defined by

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{2n}[S(Y,Z)X - S(X,Z)Y], \tag{1.10}$$

From (1.10), it follows that

$$\tilde{P}(X,Y,Z,W) = \tilde{R}(X,Y,Z,W) - \frac{1}{2n}[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)], \tag{1.11}$$

and

$$\tilde{P}(X,Y,Z,W) = g(P(X,Y)Z,W), \tag{1.12}$$

for $X,\ Y,\ Z\ ,\ W \in \chi(M)$, where S is the Ricci tensor with respect to the semi-symmetric metric connection. In fact M is projectively flat if and only if it is of constant curvature [21]. Thus the projective curvature tensor is the measure of the failure of a Riemannian manifold to be of constant curvature.

In this paper we study the projective curvature tensor on Kenmotsu manifold with respect to the semi-symmetric metric connection. The paper is organized as follows : After introduction in section 2, we give a brief account of the Kenmotsu manifolds. In section 3, we investigate the quasi-projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection and we prove that the manifold is an η-Einstein manifold. Section 4 is devoted to study ξ-projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection. Section 5 deals with ϕ-projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection. Finally, we study $P.S = 0$ in a Kenmotsu manifold with respect to the semi-symmetric metric connection.

2. Kenmotsu Manifolds

Let M be an $(2n+1)$-dimensional almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g) consisting of a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g on M satisfying [4]

$$\phi^2(X) = -X + \eta(X)\xi, \;\; g(X,\xi) = \eta(X), \tag{2.1}$$

$$\eta(\xi) = 1, \;\; \phi(\xi) = 0, \;\; \eta(\phi(X)) = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.3}$$

for all vector fields X ,Y on M. If an almost contact metric manifold satisfies

$$(D_X\phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.4}$$

then M is called a Kenmotsu manifold [14]. From the above relations, it follows that

$$D_X\xi = X - \eta(X)\xi, \tag{2.5}$$

$$(D_X\eta)(Y) = g(X, Y) - \eta(X)\eta(Y). \tag{2.6}$$

Moreover the curvature tensor K and the Ricci tensor $\tilde{S}$ of the Kenmotsu manifold with respect to the Levi-Civita connection satisfies

$$K(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$K(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.8}$$

$$K(X, \xi)Y = g(X, Y)\xi - \eta(Y)X, \tag{2.9}$$

$$\tilde{S}(\phi X, \phi Y) = \tilde{S}(X, Y) + 2n\eta(X)\eta(Y), \tag{2.10}$$

$$\tilde{S}(X, \xi) = -2n\eta(X). \tag{2.11}$$

We state the following lemma which will be used in the next section:

Lemma 2.1. [14] *Let M be an η-Einstein Kenmotsu manifold of the form $S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y)$. If $b = constant(or, a = constant)$, then M is an Einstein one.*

3. Quasi-Projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection

Definition 3.1. A Kenmotsu manifold is said to be quasi-projectively flat with respect to the semi-symmetric metric connection if

$$g(P(X,Y)Z,\phi W)=0. \tag{3.1}$$

Definition 3.2. A Kenmotsu manifold is said to be an η-Einstein manifold if its Ricci tensor $\tilde{S}$ of the Levi-Civita connection is of the form

$$\tilde{S}(X,Y)=ag(X,Y)+b\eta(X)\eta(Y), \tag{3.2}$$

where a and b are smooth functions on the manifold.

Using (1.7), (2.2) and (2.6) in (1.6), we obtain

$$\begin{aligned}R(X,Y)Z=K(X,Y)Z-3g(Y,Z)X+3g(X,Z)Y+\\ 2\eta(Y)\eta(Z)X-2\eta(X)\eta(Z)Y+\\ 2g(Y,Z)\eta(X)\xi-2g(X,Z)\eta(Y)\xi.\end{aligned} \tag{3.3}$$

Using (1.9) in (3.3), we get

$$\begin{aligned}\tilde{R}(X,Y,Z,W)=\tilde{K}(X,Y,Z,W)-3g(Y,Z)g(X,W)+3g(X,Z)g(Y,W)+\\ 2\eta(Y)\eta(Z)g(X,W)-2\eta(X)\eta(Z)g(Y,W)+\\ 2g(Y,Z)\eta(X)\eta(W)-2g(X,Z)\eta(Y)\eta(W).\end{aligned} \tag{3.4}$$

Contracting X in (3.3), we have

$$S(Y,Z)=\tilde{S}(Y,Z)-2(3n-1)g(Y,Z)+2(2n-1)\eta(Y)\eta(Z). \tag{3.5}$$

Putting $Z=\xi$ in (3.5) and using (2.11), (2.1) and (2.2), we obtain

$$S(Y,\xi)=-4n\eta(Y). \tag{3.6}$$

Again contracting Y and Z in (3.5), it follows that

$$r=\tilde{r}-2n(6n-1). \tag{3.7}$$

where r and $\tilde{r}$ are the scalar curvature with respect to the semi-symmetric metric connection and the Levi-Civita connection respectively.
Putting $X=\phi X$ and $Y=\phi Y$ in (1.11) and using (1.12), we get

$$\begin{aligned}g(P(\phi X,Y)Z,\phi W)=\tilde{R}(\phi X,Y,Z,\phi W)-\\ \frac{1}{2n}[S(Y,Z)g(\phi X,\phi W)-S(\phi X,Z)g(Y,\phi W)].\end{aligned} \tag{3.8}$$

We begin with the following:

Lemma 3.1. *Let M be a $(2n+1)$-dimensional Kenmotsu manifold. If M satisfies*

$$g(P(\phi X, Y)Z, \phi W) = 0, \quad X, Y, Z, W \in \chi(M), \tag{3.9}$$

then M is an η-Einstein manifold.

Proof: Using (3.9) in (3.8), we have

$$\tilde{R}(\phi X, Y, Z, \phi W) = \frac{1}{2n}[S(Y,Z)g(\phi X, \phi W) - S(\phi X, Z)g(Y, \phi W)]. \tag{3.10}$$

Again using (3.4) and (3.5) in (3.10), it follows that

$$\begin{aligned}\tilde{K}(\phi X, Y, Z, \phi W) = \frac{1}{n}g(Y,Z)g(\phi X, \phi W) - \frac{1}{n}g(\phi X, Z)g(Y, \phi W) - \\ \frac{1}{n}\eta(Y)\eta(Z)g(\phi X, \phi W) + \\ \frac{1}{2n}[\tilde{S}(Y,Z)g(\phi X, \phi W) - \tilde{S}(\phi X, Z)g(Y, \phi W)].\end{aligned} \tag{3.11}$$

Let $\{e_1, ..., e_{2n}, \xi\}$ be a local orthonormal basis of vector fields in M, then $\{\phi e_1, ..., \phi e_{2n}, \xi\}$ is also a local orthonormal basis. Putting $X = W = e_i$ in (3.11) and summing over $i = 1$ to $2n$, we get

$$\begin{aligned}\sum_{i=1}^{2n} \tilde{K}(\phi e_i, Y, Z, \phi e_i) = \frac{1}{n}\sum_{i=1}^{2n} g(Y,Z)g(\phi e_i, \phi e_i) - \frac{1}{n}\sum_{i=1}^{2n} g(\phi e_i, Z)g(Y, \phi e_i) - \\ \frac{1}{n}\sum_{i=1}^{2n} \eta(Y)\eta(Z)g(\phi e_i, \phi e_i) + \\ \frac{1}{2n}\sum_{i=1}^{2n}[\tilde{S}(Y,Z)g(\phi e_i, \phi e_i) - \tilde{S}(\phi e_i, Z)g(Y, \phi e_i)].\end{aligned} \tag{3.12}$$

From (3.12), we obtain

$$\tilde{S}(Y,Z) = (4n-2)g(Y,Z) - 4n\eta(Y)\eta(Z). \tag{3.13}$$

Therefore, $\tilde{S}(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z)$,

where $a = 4n - 2$ and $b = -4n$.

This result shows that the manifold is an η-Einstein manifold. This proves the Lemma .

In view of Lemma (3.1), we can state the following theorem :

Theorem 3.1. *If a Kenmotsu manifold is quasi-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an η-Einstein manifold.*

Since a and b are both constant, by Lemma (2.1), we get the following:

Corollary 3.1. *If a Kenmotsu manifold is quasi-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an Einstein manifold.*

4. ξ -Projectively flat and ϕ-Projectively flat Kenmotsu manifolds with respect to the semi-symmetric metric connection

Let C be the Weyl conformal curvature tensor of a $(2n+1)$-dimensional manifold M. Since at each point $p \in M$ the tangent space $\chi_p(M)$ can be decomposed into the direct sum $\chi_p(M) = \phi(\chi_p(M)) \oplus L(\xi_p)$, where $L(\xi_p)$ is an 1-dimensional linear subspace of $\chi_p(M)$ generated by ξ_p. Then we have a map:

$$C : \chi_p(M) \times \chi_p(M) \times \chi_p(M) \longrightarrow \phi(\chi_p(M)) \oplus L(\xi_p).$$

It may be natural to consider the following particular cases:

(1)$C : \chi_p(M) \times \chi_p(M) \times \chi_p(M) \longrightarrow L(\xi_p)$, i.e, the projection of the image of C in $\phi(\chi_p(M))$ is zero.

(2)$C : \chi_p(M) \times \chi_p(M) \times \chi_p(M) \longrightarrow \phi(\chi_p(M))$, i.e, the projection of the image of C in $L(\xi_p)$ is zero.

$$C(X,Y)\xi = 0. \tag{4.1}$$

(3)$C : \phi(\chi_p(M)) \times \phi(\chi_p(M)) \times \phi(\chi_p(M)) \longrightarrow L(\xi_p)$, i.e, when C is restricted to $\phi(\chi_p(M)) \times \phi(\chi_p(M)) \times \phi(\chi_p(M))$, the projection of the image of C in $\phi(\chi_p(M))$ is zero. This condition is equivalent to

$$\phi^2 C(\phi X, \phi Y)\phi Z = 0. \tag{4.2}$$

Here the cases 1, 2 and 3 are conformally symmetric, ξ-conformally flat and ϕ-conformally flat respectively. The cases (1) and (2) were considered in [5] and [24] respectively. The case (3) was considered in [25] for the case M is a K-contact manifold. Furthermore in [2], the authors studied contact metric manifolds satisfying (3). Analogous to the definition of ξ-conformally flat and ϕ-conformally flat, we give the folowing definitions :

Definition 4.1. A Kenmotsu manifold with respect to the semi-symmetric metric connection is said to be ξ-projectively flat if

$$P(X,Y)\xi = 0. \tag{4.3}$$

Definition 4.2. A Kenmotsu manifold is said to be ϕ-projectively flat with respect to the semi-symmetric metric connection if

$$g(P(\phi X, \phi Y)\phi Z, \phi W) = 0, \tag{4.4}$$

where $X, Y, Z, W \in \chi(M)$.

Putting $Z = \xi$ in (3.3) and using (2.1) and (2.2), it follows that

$$R(X,Y)\xi = K(X,Y)\xi + \eta(X)Y - \eta(Y)X. \tag{4.5}$$

Using (2.7) in (4.5), we obtain

$$R(X,Y)\xi = 2K(X,Y)\xi. \tag{4.6}$$

Putting $Z=\xi$ in (1.10), we have

$$P(X,Y)\xi = R(X,Y)\xi - \frac{1}{2n}[S(Y,\xi)X - S(X,\xi)Y]. \tag{4.7}$$

Using (3.6) and (4.6) in (4.7), we get

$$P(X,Y)\xi = 0. \tag{4.8}$$

Hence we can state the following theorem:

Theorem 4.1. *If a Kenmotsu manifold admits a semi-symmetric metric connection, then the Kenmotsu manifold is ξ -Projectively flat with respect to the semi-symmetric metric connection.*

Putting $Y=\phi Y$ and $Z=\phi Z$ in (3.8), we get

$$\begin{aligned} g(P(\phi X,\phi Y)\phi Z,\phi W) = g(R(\phi X,\phi Y)\phi Z,\phi W) - \frac{1}{2n}[S(\phi Y,\phi Z)g(\phi X,\phi W) - \\ S(\phi X,\phi Z)g(\phi Y,\phi W)]. \end{aligned} \tag{4.9}$$

Using (2.1), (2.2), (3.3) and (3.5) in (4.9), we have

$$\begin{aligned} g(P(\phi X,\phi Y)\phi Z,\phi W) = g(K(\phi X,\phi Y)\phi Z,\phi W) - \frac{1}{2n}[\tilde{S}(\phi Y,\phi Z)g(\phi X,\phi W) - \\ \tilde{S}(\phi X,\phi Z)g(\phi Y,\phi W)] - \frac{1}{n}[g(\phi Y,\phi Z)g(\phi X,\phi W) - \\ g(\phi X,\phi Z)g(\phi Y,\phi W)]. \end{aligned} \tag{4.10}$$

Again using (4.4) in (4.10), we obtain

$$\begin{aligned} g(K(\phi X,\phi Y)\phi Z,\phi W) = \frac{1}{2n}[\tilde{S}(\phi Y,\phi Z)g(\phi X,\phi W) - \tilde{S}(\phi X,\phi Z)g(\phi Y,\phi W)] + \\ \frac{1}{n}[g(\phi Y,\phi Z)g(\phi X,\phi W) - g(\phi X,\phi Z)g(\phi Y,\phi W)]. \end{aligned} \tag{4.11}$$

Let $\{e_1,...,e_{2n},\xi\}$ be a local orthonormal basis of vector fields in M, then $\{\phi e_1,...,\phi e_{2n},\xi\}$ is also a local orthonormal basis. Putting $X=W=e_i$ in (4.11) and summing over $i=1$ to $2n$, we get

$$\begin{aligned} \sum_{i=1}^{2n} g(K(\phi e_i,\phi Y)\phi Z,\phi e_i) = \frac{1}{2n}\sum_{i=1}^{2n}[\tilde{S}(\phi Y,\phi Z)g(\phi e_i,\phi e_i) - \tilde{S}(\phi e_i,\phi Z)g(\phi Y,\phi e_i)] + \\ \frac{1}{n}\sum_{i=1}^{2n}[g(\phi Y,\phi Z)g(\phi e_i,\phi e_i) - g(\phi e_i,\phi Z)g(\phi Y,\phi e_i)]. \end{aligned} \tag{4.12}$$

From (4.12), it follows that

$$\tilde{S}(\phi Y, \phi Z) = 2(2n-1)g(\phi Y, \phi Z). \tag{4.13}$$

Using (2.3) and (2.10) in (4.13), we obtain

$$\tilde{S}(Y, Z) = 2(2n-1)g(Y, Z) - 2(3n-1)\eta(Y)\eta(Z). \tag{4.14}$$

Therefore, $\tilde{S}(Y, Z) = ag(Y, Z) + b\eta(Y)\eta(Z)$,

where $a = 2(2n-1)$ and $b = -2(3n-1)$.
We can state the following theorem :

Theorem 4.2. *If a Kenmotsu manifold is ϕ-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an η-Einstein manifold.*

Since a and b are both constant, by Lemma (2.1), we get the following:

Corollary 4.1. *If a Kenmotsu manifold is ϕ-projectively flat with respect to the semi-symmetric metric connection, then the manifold is an Einstein manifold.*

5. Kenmotsu manifolds with respect to the semi-symmetric metric connection satisfying $P.S = 0$

In this section we consider Kenmotsu manifold with respect to the semi-symmetric metric connection M^{2n+1} satisfying condition

$$(P(U, Y).S)(Z, X) = 0$$

Then we have

$$S(P(U, Y)Z, X) + S(Z, P(U, Y)X) = 0. \tag{5.1}$$

Putting $U = \xi$ in (5.1), it follows that

$$S(P(\xi, Y)Z, X) + S(Z, P(\xi, Y)X) = 0. \tag{5.2}$$

Putting $X = \xi$ and using (3.5) and (3.6) in (1.10), we get

$$\begin{aligned} P(\xi, Y)Z = R(\xi, Y)Z - \frac{1}{2n}[\tilde{S}(Y, Z)\xi - 2(3n-1)g(Y, Z)\xi + \\ 2(2n-1)\eta(Y)\eta(Z)\xi + 4n\eta(Z)Y]. \end{aligned} \tag{5.3}$$

Again putting $X = \xi$ in (3.3) and using (2.8), we obtain

$$R(\xi, Y)Z = 2[\eta(Z)Y - g(Y, Z)\xi]. \tag{5.4}$$

Using (3.5), (3.6), (5.3) and (5.4) in (5.2), it follows that

$$\tilde{S}(Y,Z) = 2(n-1)g(Y,Z) + 2(1-2n)\eta(Y)\eta(Z). \tag{5.5}$$

Therefore, $\tilde{S}(Y,Z) = ag(Y,Z) + b\eta(Y)\eta(Z)$,

where $a = 2(n-1)$ and $b = 2(1-2n)$.
We can state the following theorem :

Theorem 5.1. *If a Kenmotsu manifold with respect to the semi-symmetric metric connection satisfying $P.S = 0$, then the manifold is an η-Einstein manifold.*

Since a and b are both constant, by Lemma (2.1), we get the following:

Corollary 5.1. *If a Kenmotsu manifold with respect to the semi-symmetric metric connection satisfying $P.S = 0$, then the manifold is an Einstein manifold.*

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Department of Mathematics, Kabi-Nazrul Mahavidyalaya, Sonamura-799181, Tripura, India.

E-mail address: ajitbarmanaw@yahoo.in

Department of Mathematics, University of Calcutta, Kolkata 700019, West Bengal, India.

E-mail address: uc_de@yahoo.com